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STRESS INTENSITY FACTOR OF A PATCHED CRACK

CHORNG-FUH LIU, HORNG-SHIAN JOU and YING-TE LEE
Department of Mechanical Engineering, National Sun Yat-Sen University, Kaohsiung, Taiwan 80424, Republic of China

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Abstract—Stress intensity factor for an isotropic cracked plate with orthotropic patch reinforcement is calculated by the finite element method. Three-dimensional (3-D) elements are employed in the present study with the help of an adhesive element analysis performed in advance. Different finite element modellings of the interface bonding are made in the present 3-D analysis to show its effect in the calculation of the stress intensity factor of a patched plate with a crack.

INTRODUCTION

Bonding an overlay over a crack can reduce the stress intensity factor significantly and is becoming a common practice for the repair of aircraft structures. Therefore, the analyses of patching effect on cracked plate have attracted the attention of some researchers (Baker and Jones, 1988; Chu and Ko, 1989; Jones and Callinan, 1979, 1981; Jones *et al.*, 1983; Keer *et al.*, 1976; Mitchell *et al.*, 1975; Ratwani, 1979 and Rose, 1982). However, almost all of these analyses, analytical or numerical, are two-dimensional (2-D). Among these researches, the most popular and practical analyses are performed with the (linear) adhesive finite element. When the stress intensity factor at the crack tip is concerned in the finite element analysis a special crack tip element is used by Jones and Callinan (1979) and Ratwani (1979). Chu and Ko (1989) use a cubic isoparametric adhesive element to analyze the same problem as in Jones and Callinan (1979) and show a good agreement. It is noted that, to form the adhesive element in these analyses, the stiffness matrices contributed from the patch and the plate are based on plane stress finite element.

Considering the 3-D nature of stresses in the patched structures, a 3-D finite element analysis should be more appropriate and is therefore conducted near the crack tip and within the patched area in the present study. To save the computing effort, a quadratic adhesive finite element (2-D) analysis proposed by Liu (1993), which is a modification of the previous linear adhesive element, is being performed first to provide the necessary boundary condition for the 3-D analysis. Also, to show the effect of modelling of the interface bonding, cases with both matched and mismatched finite element nodes at bonding interface are analyzed. Results for different thicknesses of the patches and different finite element modellings are compared to each other in the present study and to those obtained elsewhere.

PATCH-REINFORCED PLATE WITH A CRACK

Consider a plate $508 \times 635 \times 2.3$ mm with a central crack of length 38.1 mm which is subjected to uniform tensile stress 689 kPa at the ends. The boron–epoxy patch, 12.7×108 mm with different thicknesses, is stuck on both surfaces of the plate and at both tips of the crack. Due to symmetry, only a half area of the plate with a half of its thickness is analyzed

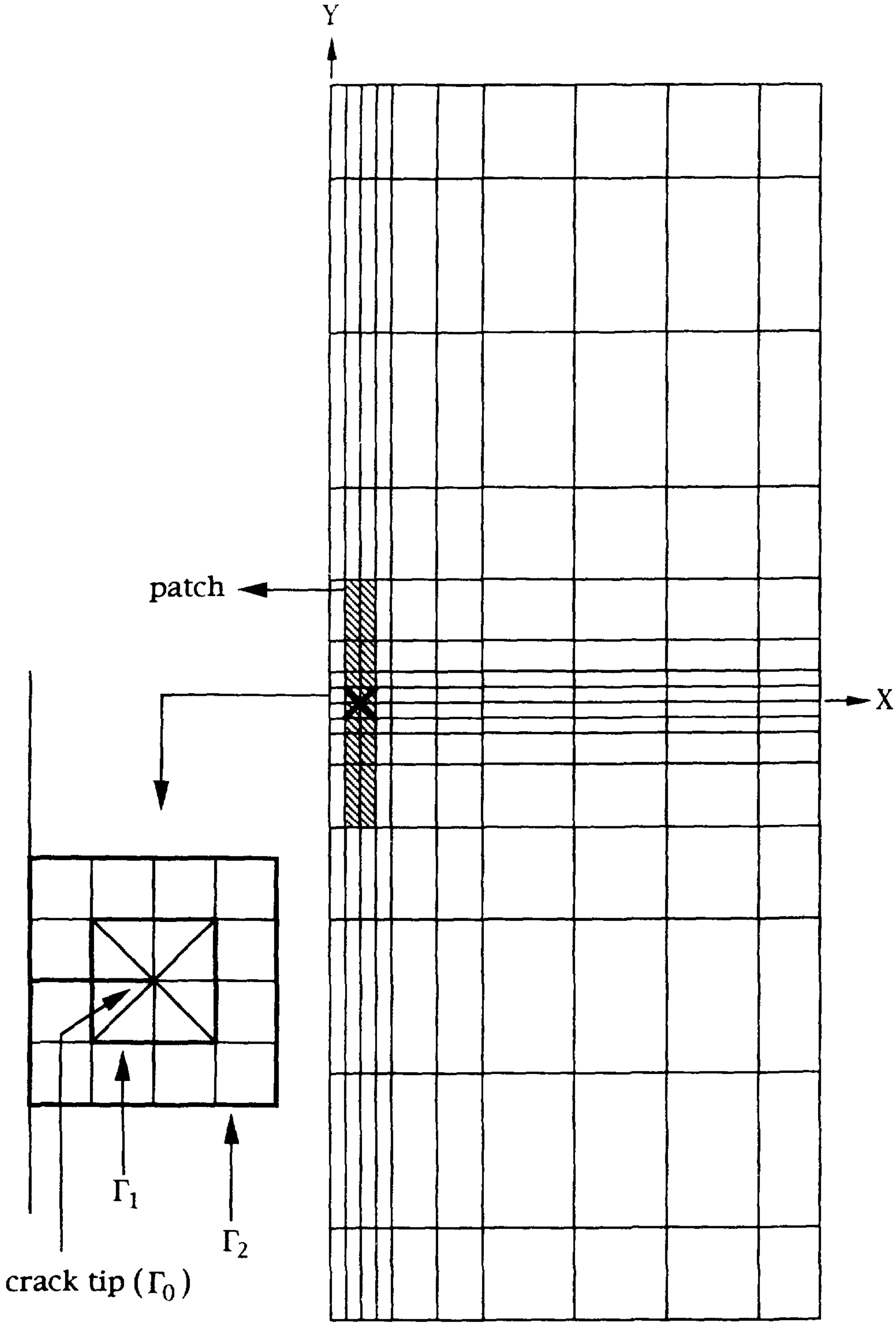


Fig. 1. Planar finite element mesh.

and is shown in Fig. 1 where it is found that the patch is parallel to the longitudinal direction and centered at the crack tip. The material properties are:

(i) plate (isotropic) $E = 71.02$ GPa, $\nu = 0.32$.
(ii) patch (orthotropic) $E_y = 208.1$ GPa, $E_y/E_x = 8.18$, $E_z = E_x$; $G_{xz} = 4.94$ GPa, $G_{yz} = 7.24$ GPa, $G_{yz} = G_{xy}$; $\nu_{yx} = 0.1677$, $\nu_{xz} = 0.035$, $\nu_{yz} = \nu_{yx}$.
(iii) adhesive (isotropic) $G = 0.965$ GPa, $\nu = 0.32$.

The thickness of the adhesive is 0.1016 mm for all the analyses. This is the same example problem as analyzed in Jones and Callinan (1979) and Chu and Ko (1989).

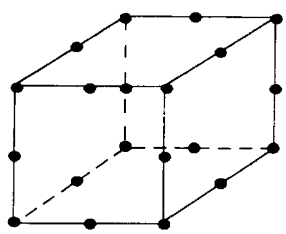
To calculate the stress intensity factor at the crack tip, two approaches are employed in the present investigation, one is to use the quarter-point crack tip singular element proposed by Barsoum (1977) (the displacement approach) to model the area or space around the crack tip, the other is to use the energy release rate proposed by Parks (1974) and Hellen (1975) (the energy approach).

A 2-D analysis is performed first with the mesh shown in Fig. 1. The results obtained are then used as the boundary conditions for the subsequent 3-D analysis which is performed only for a part of the patched area enclosed by Γ_2.

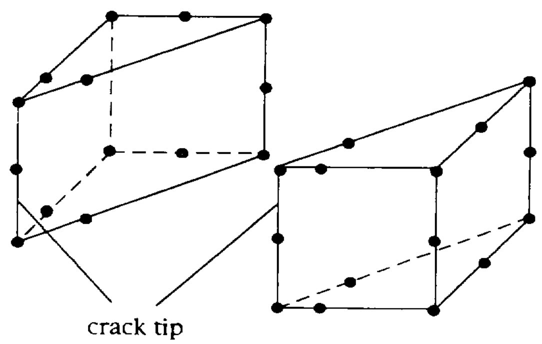
When using the displacement approach, K_I at a particular z-coordinate can be obtained from the nodal displacements on each element edge (except the one on the x-axis) with the same z-coordinate and emanating from the crack tip by the following formula.

$$K_I[(2\kappa+1)\sin(\theta/2)-\sin(3\theta/2)] = 4G_s(2\pi/d)^{1/2}(4V_B-V_C-3V_A)$$

where $\kappa = (3-\nu)/(1+\nu)$ and ν is Poisson's ratio of the plate. It can thus be recognized that we are computing the stress intensity factor for plane stress cases. V_A, V_B and V_C are the three nodal displacements in the y-direction at the crack tip, at the quarter-point position and at the other end of the element edge. d is the length of the element edge. θ is the angle between the element edge and the x-axis, and is taken as zero along the x-axis and positive when counterclockwise. The obtained values of K_I for different element edges are averaged to give the stress intensity factor for the present problem. With the energy approach, Γ_0 is selected to be the point of the crack tip and Γ_1 is the outside boundary of the eight prism elements around the crack tip, see Figs 1–3. The virtual crack extension is $\delta L = 10^{-5}\times L$, L is the original crack length. K_I is calculated by



(a) patch and adhesive



(b) plate

Fig. 2. Inconsistent 3-D modelling: (a) conventional brick element in the adhesive and the patch; (b) quarter-point prism element in the plate.

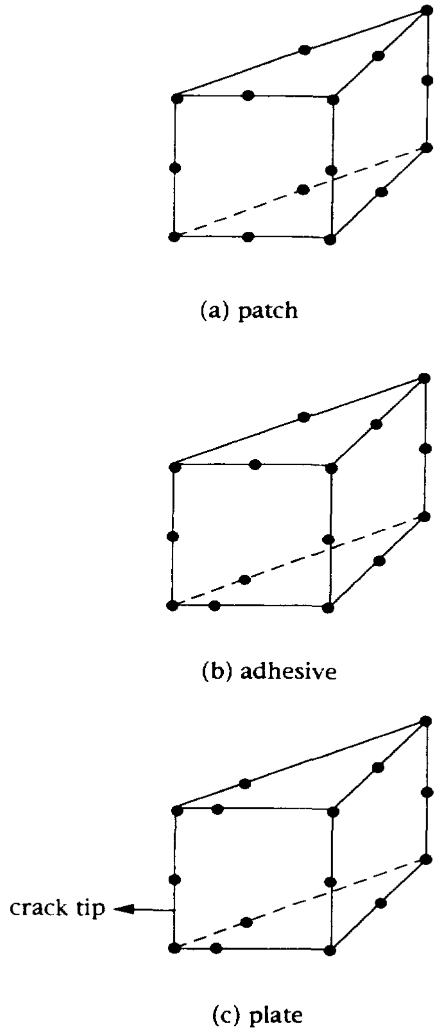


Fig. 3. Consistent 3-D modelling: (a) conventional prism element in the patch; (b) prism element in the adhesive, some midside nodes shifted; (c) quarter-point prism element in the plate.

$$\frac{K_{\mathrm{I}}^2}{E} = -\frac{1}{2}\{\Delta\}^{\mathrm{T}}\left[\frac{1}{\delta L}\sum_{i=1}^{N_c}([k_i]_2-[k_i]_1)\right]\{\Delta\}$$

where $[k_i]_1$ are the stiffness matrices of the elements enclosed by Γ_0 and Γ_1 with initial crack length and $[k_i]_2$ are those when the crack length is increased by δL. N_c is the number of elements between Γ_0 and Γ_1. $\{\Delta\}$ is the obtained displacement vector when the crack is not extended. See Parks (1974) and Hellen (1975) for more details about the energy approach.

RESULTS AND DISCUSSIONS

In the present study, domain Γ_2 is divided into 20 solid elements (eight prismatic quarter-point crack tip elements and 12 conventional quadratic brick elements) and each of the patch and the adhesive is modelled with 16 conventional quadratic brick elements. Figure 2 shows the prism elements in the plate and the corresponding brick element in the adhesive and the patch layers. It is noted that there is only one layer of elements in each of the plate, the patch and the adhesive.

Two values of K_{I} are obtained in the displacement approach, one is the K_{I} at the plate-adhesive interface, the other is at the midplane of the plate. As for the energy approach, the whole thickness of the crack front is assumed to extend the same δL. Only one value of K_{I} is obtained for the energy approach.

Table 1. Stress intensity factors for 3-D analyses with inconsistent modelling

Thickness of patch (mm)	Displacement approach K_I (KPa·m$^{1/2}$) Midplane of plate	Displacement approach K_I (KPa·m$^{1/2}$) Plate-adhesive interface	Energy approach K_I	Jones (1979) K_I	Arin (1974) Davis (1978) K_I
0.127	65.1	73.4	79.1	77.8	75.4
0.254	51.2	57.7	59.3	65.0	61.4
0.381	36.2	45.0	48.8	57.9	51.9
0.508	30.4	38.8	42.0	53.1	44.7
0.762	23.8	31.2	33.8	46.7	31.3

Table 2. Stress intensity factors for 3-D analyses with consistent modelling

Thickness of patch (mm)	Displacement approach K_I (midplane of plate)	Displacement approach K_I (plate-adhesive interface)	Energy approach K_I
0.127	55.5	54.4	66.5
0.254	37.7	38.0	48.3
0.381	29.1	30.0	39.1
0.508	24.0	25.3	33.4
0.762	18.2	19.9	26.8

Table 1 shows the results of the present 3-D analysis. It should be noted that inconsistency exists in the present finite element modelling, i.e. there is mismatching of the midside nodes at the plate-adhesive interface, see Fig. 2. In that case, some area in the plate-adhesive interface around the crack tip is assumed to be debonded and a higher stress intensity factor is likely to occur.

To have a more realistic modelling, each of the brick elements in the patch and in the adhesive near the crack tip are divided into two prism elements as shown in Fig. 3, the prism elements in the patch have the midside nodes located in the middle position of the edges, as regular ones do. However, for the prism elements in the adhesive, the midside nodes on the plate-adhesive interface are shifted to the quarter-point position near the crack tip to match with those in the plate while on the patch-adhesive interface, nodes of the prism element in the adhesive coincide with those in the patch. No more debonding exists in this modelling.

The results of this consistent model, as shown in Table 2, change significantly from those of the inconsistent 3-D model in Table 1. The matching of midside nodes has an obvious effect to constrain the deformation of the plate and to lower the stress intensity factor. Compared to our results of the 3-D consistent model, those shown in Arin (1974), Davis (1978) and Jones and Callinan (1979) overestimate the stress intensity factors (see Table 1) and may represent conservative analyses. Also, it can be observed that the values of K_I decrease monotonically along with the increase of the patch thickness as found before and that the stress intensity factor calculated by the energy method is always higher than those by the displacement approach.

CONCLUSIONS

The stress intensity factor of a cracked isotropic plate with an orthotropic patch is determined by the finite element technique. Two-dimensional finite element analyses are performed first with the patch region being modelled with the adhesive element. Part of the results from 2-D analyses are then used as the boundary conditions for the following 3-D analyses which are conducted only for a small space of the patched plate near the crack tip. Quarter-point crack tip elements are used to model the crack tip region in the plate. Cases with both matched and mismatched midside nodes along the element edges on the plate-adhesive interface are considered. The stress intensity factors are calculated from the displacement and the energy release rates which are derived by the displacement approach and the energy approach, respectively.

From the results of both the matched and mismatched cases in the present study, it may be concluded that a realistic finite element modelling plays an important role in the analysis of patched crack. Compared to the present consistent matched 3-D results, those obtained previously by a whole 2-D analysis were overestimated and the difference increases as the patch thickness increases. Due to the assumptions and approximations employed in the 2-D analysis and the 3-D nature of stresses near the patched crack tip, the present 3-D consistent model is more realistic and may serve as an alternative of calculating the stress intensity factor of a patched crack and should be considered in critical cases.

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